A short guide for small area estimation in household surveys

Illustration to poverty mapping in Palestine with expenditure survey and census data

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Palestinian Expenditure Consumption Survey (PECS) 2016/17

- Sample size: $n = 18,363$ persons out of $N = 4,266,953$ (43 out of 10,000)
- Sample sizes of regions by gender are fine:

	Gaza	West Bank	Total
Women	2569	6550	9119
Men	2578	6666	9244
Total	5147	13216	18363

Palestinian Expenditure Comsumption Survey (PECS) 2016/17

- What if we wish to estimate at **local** level?
- 315 **localities** in census: 162 in PECS, 157 **unsampled**.
- Sample sizes localities by gender:

	Min	Q1	Median	Mean	Q3	Max
Women	14	26	35	56.29	61.5	405
Men	13	28	36	57.06	63	464

SMALL AREA ESTIMATION

- **Areas/domains:** Subpopulations of interest (Example: Localities $\times$ gender).
- **Direct estimator:** Based **only** on the survey data **from** the target area/domain.
- **Small area:** Area/domain for which the considered **direct** estimator of the target indicator has **unacceptable** sampling error.

INDIRECT ESTIMATION

- The idea is **borrowing strength** from the other areas.
- Use auxiliary data sources (**census** or other, ideally a census) that contains some variables related to our target variable and observed also in the survey.
- Consider that the target variable is related with the auxiliary variables **similarly** for all the areas (regression model).
- Include random **area** effects to account for unexplained between-area **heterogeneity** (mixed regression model).

MODEL-BASED ESTIMATION

- Fit the model to the survey data from **all** the areas.
- Total survey sample size is typically large, so borrowing a lot of strength.
- Use the fitted model to estimate in the small areas.
- Efficiency gains can be substantial.

NOTATION

- Num. areas/domains: D
- Area popn. sizes: $N_1, \ldots, N_D$
- Area index: $d = 1, \ldots, D$
- Unit index: $j = 1, \ldots, N_d$
- **Welfare** measure for indiv. j in domain d: E_{dj}
  (Example: monthly household expenditure per adult equivalent)
- Poverty line: z

POVERTY INDICATORS

- Poverty **rate**:

$$F_{0d} = \frac{1}{N_d} \sum_{j=1}^{N_d} \underbrace{I(E_{dj} < z)}_{= \begin{cases} 1 \text{ if } E_{dj} < z; \\ 0 \text{ otherwise} \end{cases}}$$

- Poverty **gap**:

$$F_{1d} = \frac{1}{N_d} \sum_{j=1}^{N_d} \underbrace{\left(\frac{z - E_{dj}}{z} \right)}_{\text{Rel. distance to pov. line}} I(E_{dj} < z)$$

- In general, **FGT** indicators: For $\alpha \geq 0$,

$$F_{\alpha d} = \frac{1}{N_d} \sum_{j=1}^{N_d} \left(\frac{z - E_{dj}}{z} \right)^{\alpha} I(E_{dj} < z).$$

✓ *Foster, Greer & Thornbecke (1984), Econometrika.*

EB ESTIMATORS

- The distribution of expenditures E_{dj} is highly right skewed.
- We need to take some transformation to achieve Normality: $y_{dj} = \log(E_{dj} + k)$
- We consider a model with random area effects for y_{dj}:

$$y_{dj} = \mathrm{x}'_{dj}\boldsymbol{\beta} + u_d + e_{dj}, \quad j = 1, \ldots, N_d, \; d = 1, \ldots, D$$
$$u_d \overset{iid}{\sim} N(0, \sigma_u^2), \quad e_{dj} \overset{iid}{\sim} N(0, \sigma_e^2)$$

- We obtain the **empirical best** (EB) predictor of the target indicator for each area of interest:

$$\hat{F}_{\alpha d}^{EB} = E[F_{\alpha d}|\mathrm{y}_s; \hat{\boldsymbol{\theta}}]$$

✓ *Molina & Rao (2010), Can. J. Stat.*

DATA DESCRIPTION

- **Data:** Palestinian Expenditure Consumption Survey (PECS) from 2016/2017 and Population Census from 2017.
- **Target:** Estimate poverty rates and gaps for Palestinian localities by gender.
- **Areas:** In census, 319 **localities**$\rightarrow D = 162$ in survey. We compute estimates for each **sampled** locality by gender.
- **Welfare measure:** E_{dj} monthly expenditure per adult equivalent (ILS).
- **Poverty line:** $z = 10,027$ ILS $\rightarrow$ approx. **26 %** popn. below pov. line.

FITTED MODEL

- We fit a separate model for each gender.
- **Explanatory variables:**
  - ✓ Indicators of region (Gaza, West Bank), type of locality (rural/urban, camp).
  - ✓ Household characteristics (size, prop. females, employed ratio).
  - ✓ Household head characteristics (unemployed, employisrasett, employnatgov, refugstat, diff, neverschool, secondabove).
  - ✓ Dwelling characteristics (type, tenure, num. rooms).
  - ✓ Supplies (water, waste, heating systems, freezer, etc.)

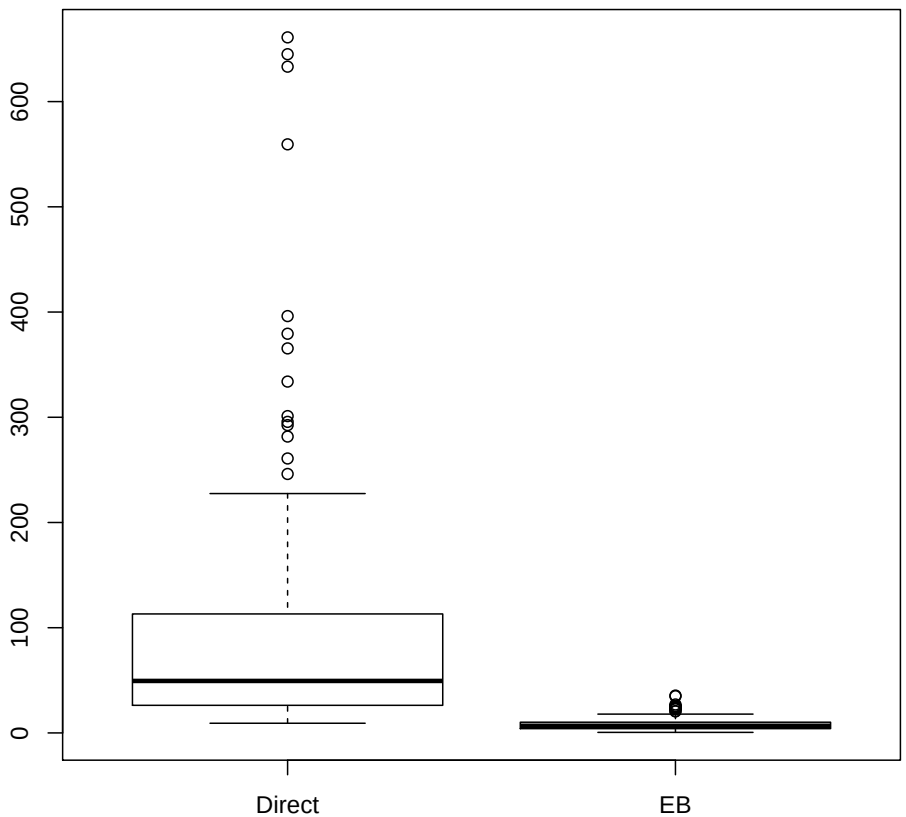
MODEL CHECKING

- Model coefficients take reasonable signs.
- All covariates with significant categories for both genders.
- **Explanatory power:** $R^2 = 53.6\,\%$, both genders.
- Data indicates nothing against normality of model residuals, linearity, heteroscedasticity. Model seems to fit well.

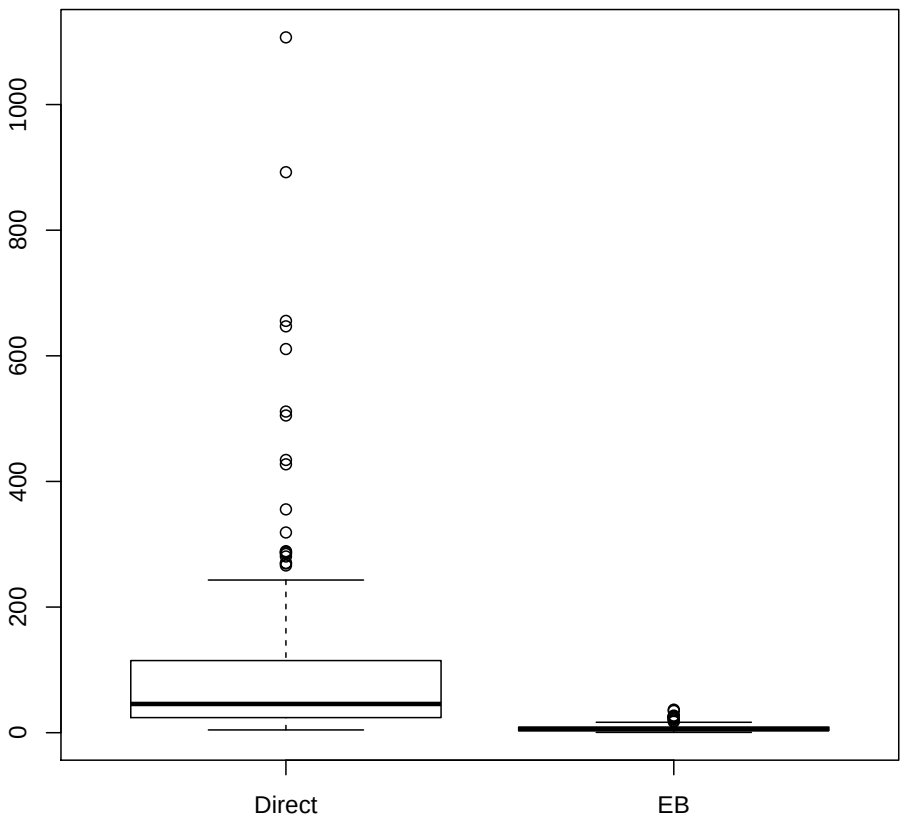
QUALITY EB vs. DIRECT: POV. RATE

- ✓ Median MSE Women: Direct **47**, EB: **6.7**
- ✓ Median MSE Men: Direct **45.8**, EB: **5.5**

MSE: Women

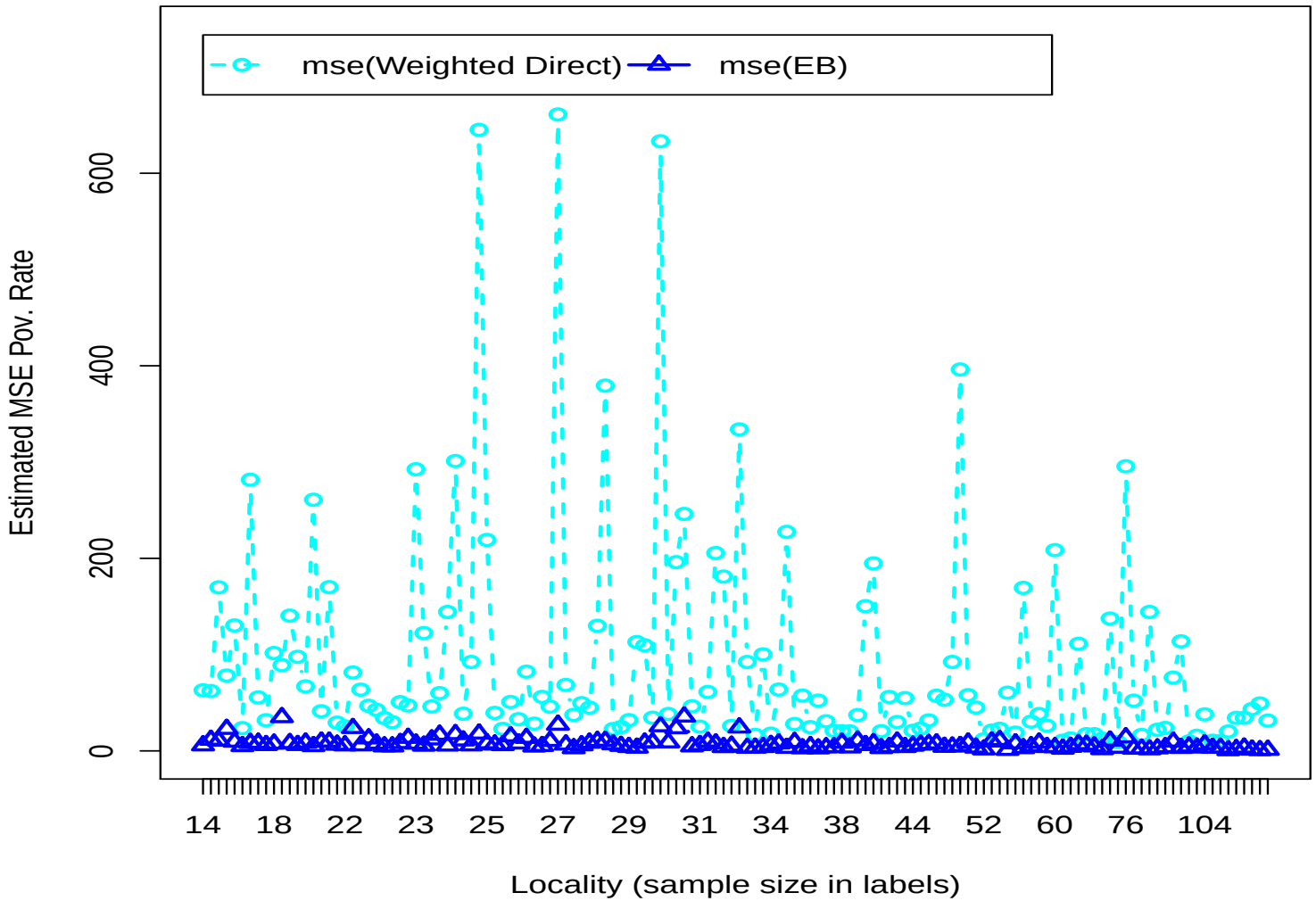


MSE: Men

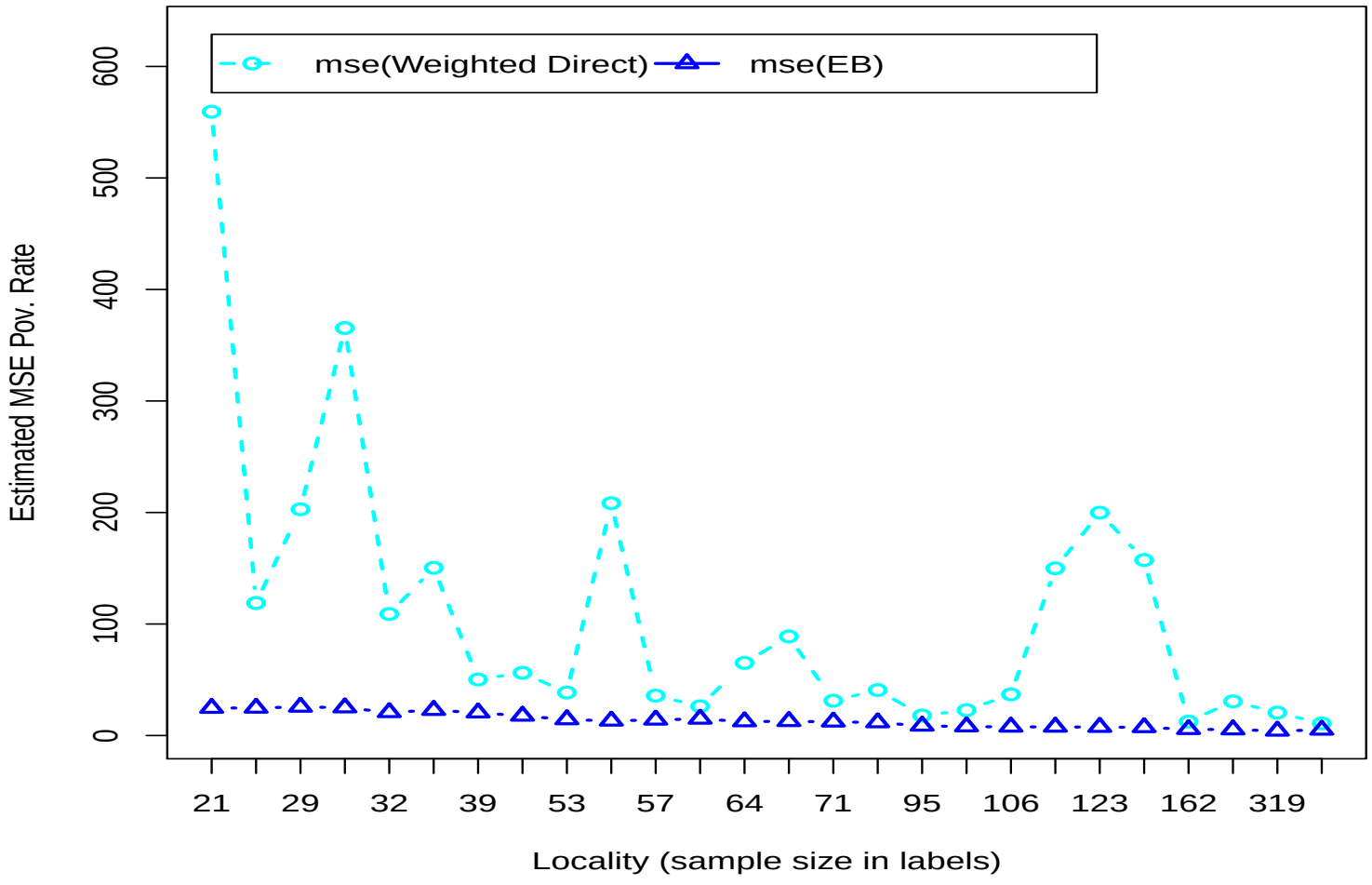


EB vs. DIRECT: WOMEN, WEST BANK

✓ Reduction in **all** but one locality, **84 %** average MSE reduction!



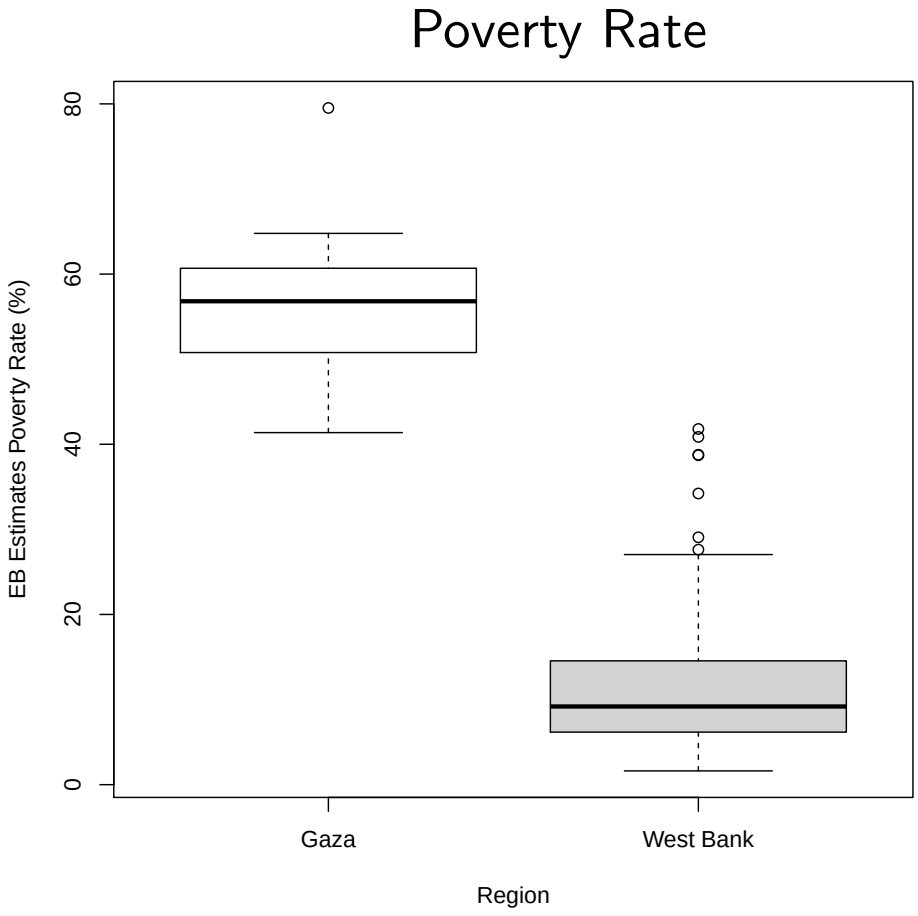
EB vs. DIRECT: WOMEN, GAZA

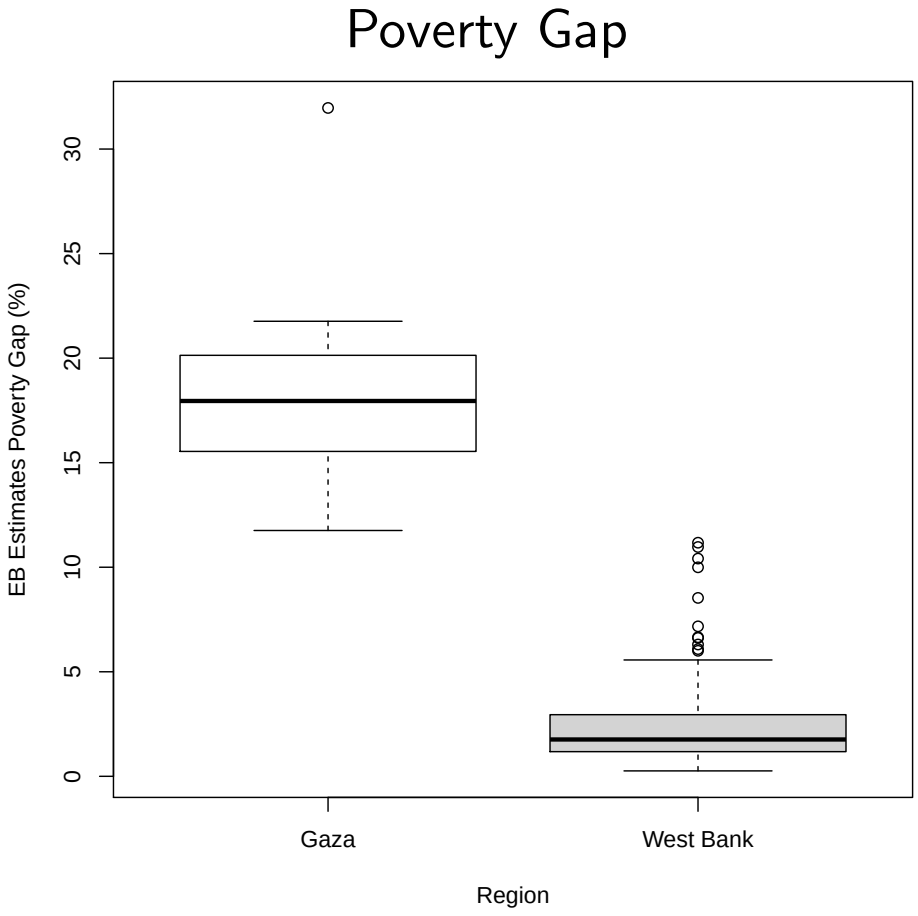


✓ **Great gains** also for Pov. **Gap** (not shown)!

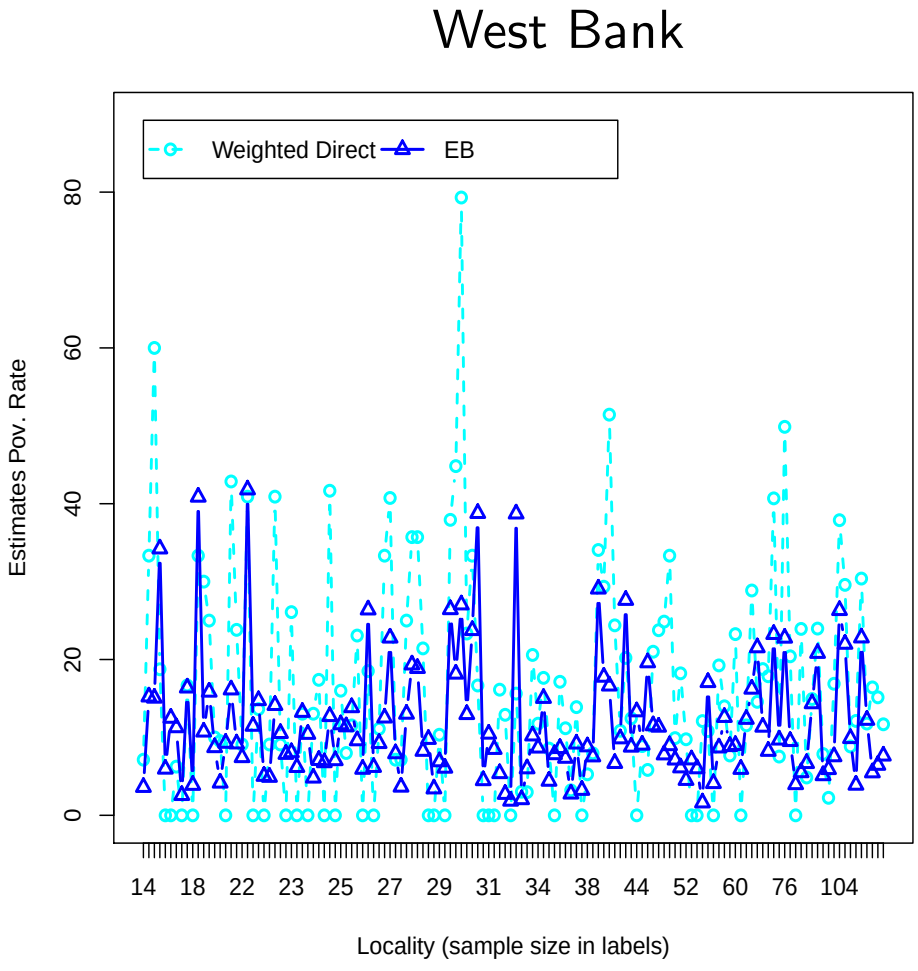
ESTIMATES BY REGION

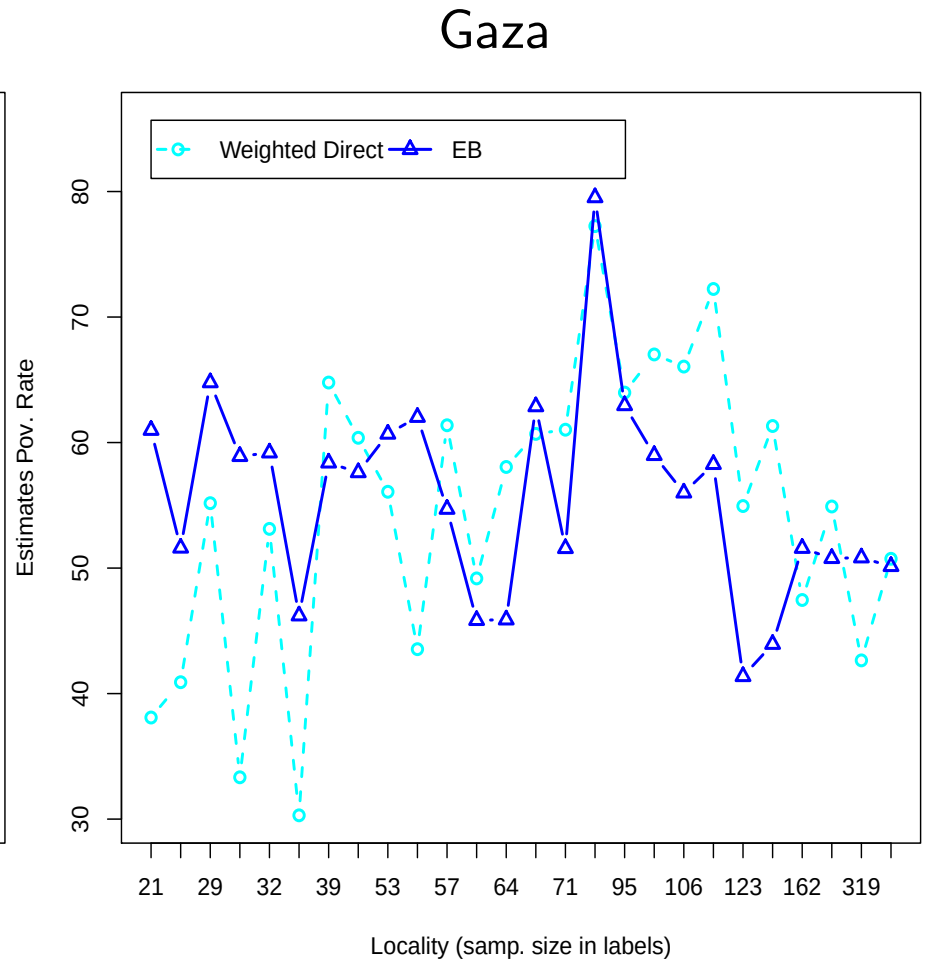
- ✓ Median Pov. Rate: Gaza **55 %**, West Bank: **8.3 %**
- ✓ Median Pov. Gap: Gaza **17.4 %**, West Bank: **1.5 %**





ESTIMATED POV. RATE: WOMEN





CONCLUSIONS

- The use of **census data** in a model allows us to obtain disaggregated estimates of **much better quality**.
- **Direct** estimates equal to **zero** for many localities (32 for Men, 29 for Women) and **highly unstable**.
- **EB** estimates **never zero** and much more stable (smooth), without visible systematic design bias.
- **The reduction of error** of model-based estimators with respect to direct estimators is notorious (over 82 %).

CONCLUSIONS

- The considered unit level methodology allows to disaggregate **at any desired level**.
- We can estimate **whatever indicator** that is function of expenditure.
- The considered model **fits rather well** these data. Still, model variations are being explored.
- Gaza has **much larger** pov. rates and gaps. Perhaps using a different pov. line.
- No great differences between men and women, although women with slightly greater estimates for about 70 % of localities in West Bank.

- ✓ MANY THANKS TO UN-ESCWA AND PCBS FOR GREAT DATA PREPARATION!
- ✓ THANK YOU ALL FOR YOUR ATTENTION!