WELCOME

Session 8

# Functional forms in CGE models

# Choosing the functional forms

The major constraints in applied models is that the functional forms have to be consistent with the theoretical approach (well behaved).

A functional form is well behaved means: non negative, non decreasing, continuous and quasi-concave in each point. Furthermore, for the demand functions, they have to be homogenous of degree zero and result in a system of demand in conformity with the Walras law.

This explains why in CGE models, the most used forms are: Cobb-Douglas, CES, CET and LES.

# The C-D function in the case of the consumer theory

With a Cobb-Douglas utility function, the consumer's demand is obtained as the solution of the following maximization program:

$$
\begin{aligned}
&Max\, U = \prod C_i^{\alpha_i} \\
&s.t. \sum_i P_i . C_i = R \quad and \quad \sum_i \alpha_i = 1
\end{aligned}
$$

The demand of each commodity i reads:

$$C_i = \frac{\alpha_i . R}{P_i}$$

Main characteristics: Price and income elasticity, as well as the elasticity of substitution between each pair of goods, are all equal to one whereas the cross price elasticity is nil.

Despite these strong assumptions, many authors resort to the C-D function given that it can be easily calibrated and don't require outside estimated of the free parameters.

With a Cobb-Douglas utility function, the only unknown parameter is the budgetary share of the consumption of each commodity in overall consumption.

Considering the income, consumption and prices provided by the SAM, the computation of the share of each good in overall consumption income (total expenditures) is a simple inversion of the demand equations.

$$\alpha_i = \frac{C_i . P_i}{R}$$

However, such restrictions are never observed in empirical estimates
In order to relax some of these restrictions, one may choose some more flexible functional forms.

# The CES function in the case of the consumer theory

The program of the consumer can be written as follows:

$$\max U = \left[b_1 q_1^{-\rho} + b_2 q_2^{-\rho}\right]^{-1/\rho} = \left[b_1 q_1^{\frac{\sigma-1}{\sigma}} + b_2 q_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$s.t\ R = p_1 q_1 + p_2 q_2$$

With $\sigma = \dfrac{1}{1+\rho}$ and $\sigma = \dfrac{\partial \ln(X_i / X_j)}{\partial \ln(P_j / P_i)}$

If $\sigma$= 0, then the products are said perfect complements and if $\sigma$ = infinity, then the products are said perfect substitutes. If between these two values, the products are imperfect substitutes.

The resulting demand functions can be written as follows

$$q_1 = \frac{b_1^{\sigma} p_1^{-\sigma} R}{\left[b_1^{\sigma} p_1^{1-\sigma} + b_2^{\sigma} p_2^{1-\sigma}\right]} \qquad q_2 = \frac{b_2^{\sigma} p_2^{-\sigma} R}{\left[b_1^{\sigma} p_1^{1-\sigma} + b_2^{\sigma} p_2^{1-\sigma}\right]}$$

The CES function avoids the unit price elasticity constraint imposed by the C-D function, has a unitary income elasticity as in the C-D case and implies a constant elasticity of substitution between each two commodities.

The CES function is the most commonly used function for modeling international trade in CGE models

Two parameters are to be calibrated: the share parameters and the elasticity of substitution. All the other information is provided by the SAM.

In the practice, we introduce exogenously the elasticity of substitution from econometric studies if possible, or from other studies on countries similar in terms of characteristics.

Once the elasticity of substitution is available, the share parameter can be deduced straightforward.

# The Linear expenditures system

The Stone-Geary function also known as LES does not assume unit income elasticity. The consumer is assumed to maximize his utility function according to his income R

$$U = (q_1 - \theta_1)^{\beta_1} (q_2 - \theta_2)^{\beta_2}$$

The solution of this optimization problem leads to the demand functions:

$\beta_1$ and $\beta_2$ are called the marginal budget shares: $\frac{\partial(p_i q_i)}{\partial R} = \beta_i$

$$p_1 q_1 = p_1\theta_1 + \beta_1\left[R - p_1\theta_1 - p_2\theta_2\right]$$

$$p_2 q_2 = p_2\theta_2 + \beta_2\left[R - p_1\theta_1 - p_2\theta_2\right]$$

These two equations can be interpreted as follows: the expenditures of the consumer are divided in two components: the first component, represented by $P_1\vartheta_1$ and $P_2\vartheta_2$ constitutes the minimal expenditure allocated by the consumer in order to ensure a minimum of subsistence. The second component is the supernumerary income which is allocated to the consumption of the two goods according to the budget share parameters.

The parameters that are unknown and have to be calibrated are $\vartheta$ and β

Two possibilities exist for the calibration of these two parameters:

- In case that the data are available, we can estimate them econometrically

- In case the data are unavailable, we proceed by:

The marginal budget share parameters can be derived from the expression of the income elasticity which has to be known:

$$e_{iR} = \left(\frac{\partial q_i}{\partial R}\right)\left(\frac{R}{q_i}\right) = \beta_i\left(\frac{1}{p_i}\right)\left(\frac{R}{q_i}\right) = \frac{\beta_i R}{p_i q_i}$$

The consumption of subsistence can be derived using the Frisch coefficient:

$$\omega = \left(\frac{\partial \lambda}{\partial R}\right)\left(\frac{R}{\lambda}\right) = -\frac{R}{R - \sum_i p_i q_i}$$

$\lambda$ represents the marginal utility of income ($\partial U/\partial R$) and $\omega$ the Frisch coefficient

Frish stipulates that for a development economy, the consumption of subsistence is high, so that the supernumerary income is weak and the Frisch coefficient is close to -2. For a developed economy, the Frisch coefficient is close to -1.

Having the Frisch coefficient, we estimate the supernumerary income and then the consumption of subsistence.


THANK
YOU