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Overview of the model construction for Tunisia

1. Prices Definition:

Import Price:

The import price in LCU (local-currency units) is the price paid by domestic users for imported commodities (exclusive of the sales tax). It is a transformation of the world price of these imports, considering the exchange rate and import tariffs plus transaction costs (the cost of trade inputs needed to move the commodity from the border to the demander) per unit of the import.

$$PIMP_{C,R,t} = pwm_{C,R,t} \cdot \left(1 + tm_{C,R,t}\right) \cdot EXR_t + \sum_{CT}(PDEM_{CT,t} \cdot icm_{CT,C})$$

Prices Definition:

The export price: in LCU is the price received by domestic producers when they sell their output in export markets. The tax and the cost of trade inputs reduce the price received by the domestic producers of exports. The domain of the equation is the set of exported commodities, all of which are produced domestically.

$$PEXP_{C,R,t} = pwe_{C,R,t} \,.\left(1 - te_{C,R,t}\right).EXR_t - \sum_{CT}(PDEM_{CT,t} \,.ice_{CT,C})$$

Demand Price of Domestic Non traded Goods: The model includes distinct prices for domestic output that is used domestically. In the presence of transaction costs, it distinguishes between prices paid by demanders and those received by suppliers.

$$PD_{C,t} = PDS_{C,t} + \sum_{CT}(PDEM_{CT,t} \,.icd_{CT,C})$$

Prices Definition:

Absorption: defined as total domestic spending on a commodity at domestic demander prices. Absorption is expressed as the sum of spending on domestic output and imports at the demand prices, *PDD* and *PM*. The prices *PDD* and *PM* include the cost of trade inputs but exclude the commodity sales tax.

$$PDEM_{C,t}\left(1 - TQ_{C,t}\right) DEM_{C,t} = PD_{C,t}.D_{C,t} + \sum_R (PIMP_{C,R,t}.IMP_{C,R,t})$$

Marketed Output Value: For each domestically produced commodity, the marketed output value at producer prices is stated as the sum of the values of domestic sales and exports. Domestic sales and exports are valued at the prices received by the suppliers, PDS and PEXP, both of which have been adjusted downwards to account for the cost of trade inputs.

$$PYC_{C,t}\,.YC_{C,t} = PDS_{C,t}.D_{C,t} + \sum_R \left(PEXP_{C,R,t}.EXP_{C,R,t}\right)$$

Prices Definition:

Output Price: The gross revenue per activity unit, the activity price, is the return from selling the output or outputs of the activity, defined as yields per activity unit multiplied by activity-specific commodity prices, summed over all commodities. This allows for the fact that activities may produce multiple commodities.

$$PYA_{A,t} = \sum_C (PYAC_{A,C,t} . \theta_{A,C})$$

Price of aggregate intermediate input: The activity-specific aggregate intermediate input price shows the cost of disaggregated intermediate inputs per unit of aggregate intermediate input. It depends on composite commodity prices and intermediate input coefficients, which show the quantity of input commodity *c* per unit of aggregate intermediate input.

$$PINTA_{A,t} = \sum_C (PDEM_{C,t} . ica_{A,C})$$

Prices Definition:

Value-added Price: For each activity, total revenue net of taxes is fully exhausted by payments for value-added and intermediate inputs

$$PVA_{A,t}.VA_{A,t} = PYA_{A,t}.(1 - ta_{A,t}).YA_{A,t} - PINTA_{A,t}.INT_{A,t}$$

Consumer Price Index:

$$CPI_t = \sum_C cwts_C.PDEM_{C,t}$$

Domestic Producer Price Index:

$$DPI_t = \sum_{CD} dwts_{CD}.PDS_{CD,t}$$

GDP Definition: The Gross Domestic Product is the sum of the gross value added by all resident producers in the economy.

$$GDP_t = \sum_A VA_{A,t}$$

2. Production Block:

- Production is carried out by activities that are assumed to maximize profits subject to their technology, taking prices (for their outputs, intermediate inputs, and factors) as given.
- It acts in a perfectly competitive setting.
- The CGE model includes the first-order conditions for profit-maximization by producers. Producers choose the optimal bundle between values added and aggregated intermediate inputs, which is modeled by the Leontief function.

2. Production Block:

The production and trade block covers four categories:

(i) domestic production and input use;

(ii) the allocation of domestic output to home consumption, the domestic market, and exports;

(iii) the aggregation of supply to the domestic market (from imports and domestic output sold domestically); and

(iv) (iv) the definition of the demand for trade inputs that is generated by the distribution process.

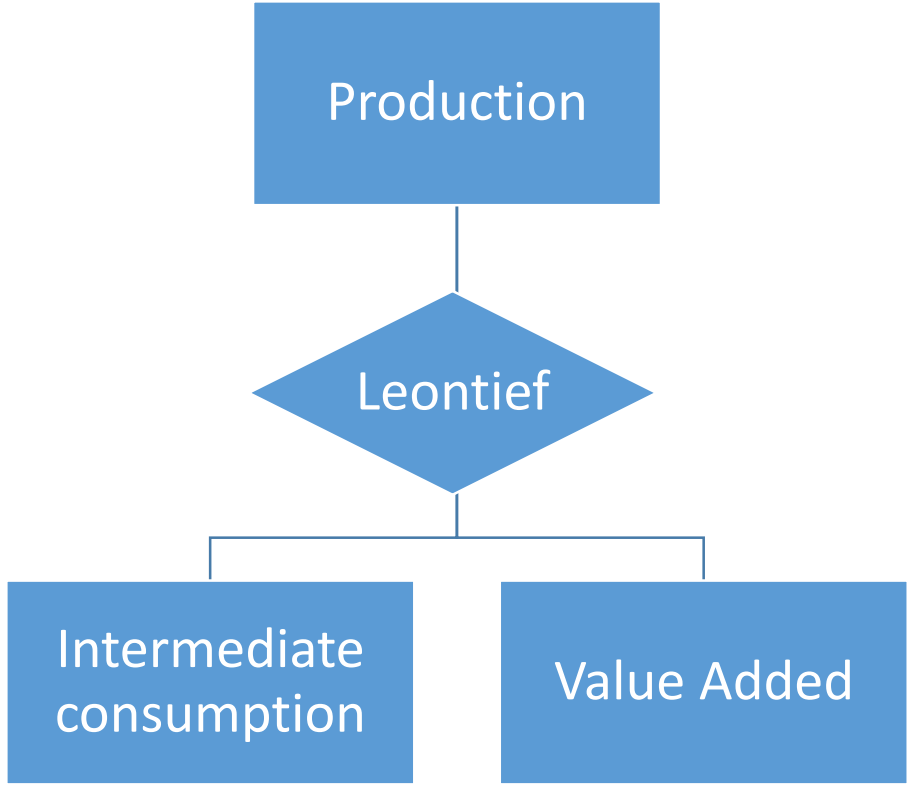
2. Production Block:

Demand for Aggregated Intermediate Input

$$INT_{A,t} = inta_A \,.\, YA_{A,t}$$

Demand for Aggregate Value-Added

$$VA_{A,t} = iva_A \,.\, YA_{A,t}$$



2.1. Value-Added and Factor Demands :

Capital Demand

$$K_{A,t}^{AGG} = VA_{A,t} \cdot A_{A,t}^{va\,(\sigma_A^{va}-1)} \cdot \left(a^{va} \cdot \frac{PVA_{A,t}}{PK_{A,t}^{AGG}}\right)^{\sigma_A^{va}}$$

Aggregated Labor Demand:

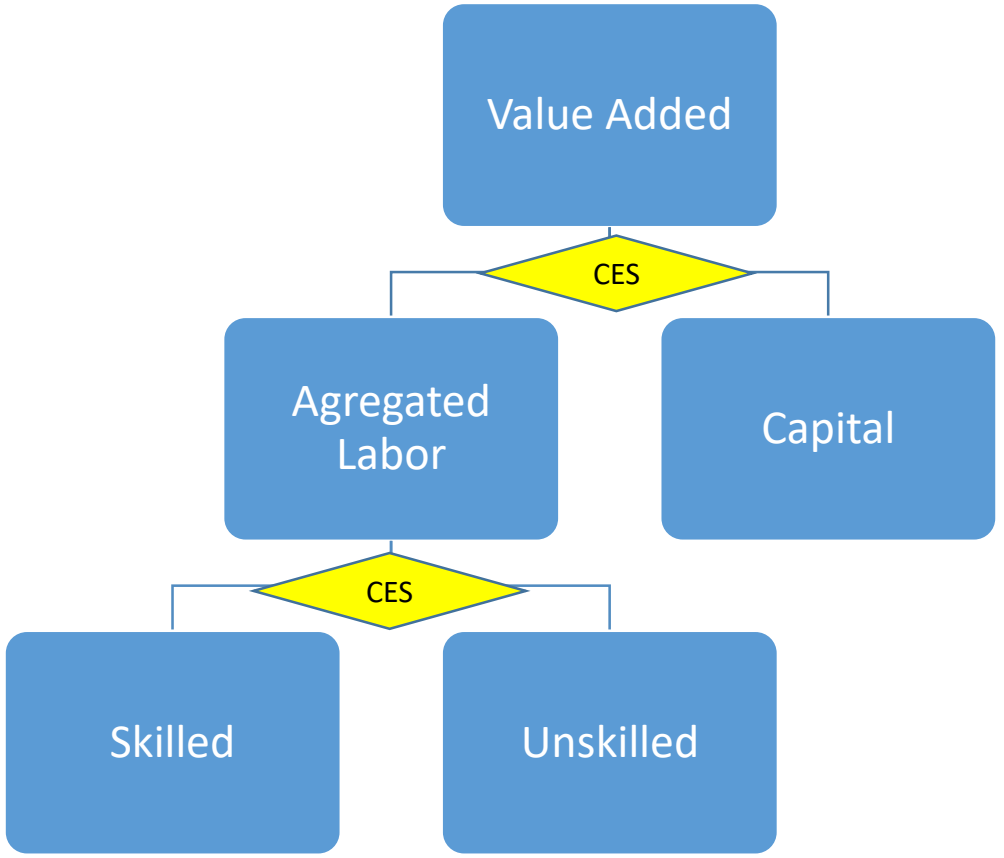
$$L_{A,t}^{AGG} = VA_{A,t} \cdot A_{A,t}^{va\,(\sigma_A^{va}-1)} \cdot \left(b^{va} \cdot \frac{PVA_{A,t}}{W_{A,t}^{AGG}}\right)^{\sigma_A^{va}}$$

Skilled labor demand

$$L_{SKL\,(A,t)} = L\_AGG(A,t)\left(b\_L\_AGG(A)\frac{W\_AGG(A,t)}{W\_SKL(A,t)}\right)^{\sigma^{AGG\_L}(A)}$$

Unskilled labor demand

$$L_{UNS\,(A,t)} = L\_AGG(A,t)\left(b\_L\_AGG(A)\frac{W\_AGG(A,t)}{W\_UNS(A,t)}\right)^{\sigma^{AGG\_L}(A)}$$



2.2 Factor market equilibrium:

Value Added Equilibrium

$$PVA_{A,t} \,.VA_{A,t} = PK_{A,t}^{AGG} \,.K_{A,t}^{AGG} + W_{A,t}^{AGG} \,.L_{A,t}^{AGG}$$

Labor market equilibrium

$$W_{A,t}^{AGG} = W_t^{Bar}$$

$$\sum_A \mathrm{L}_{\mathrm{A,t}}^{\mathrm{AGG}} \;\; (1 + \mathrm{UNEMP}_{\mathrm{t})} = \mathrm{L}_{\mathrm{t}}^{\mathrm{tot}}$$

Capital Market equilibrium

$$K_{capital\ ,A,t} = K_{A,t}^{AGG} \; (b^{K_{capital\ ,A}^{AGG}} . \frac{PK_{A,t}^{AGG}}{rK_{capital\ ,A,t}})^{\sigma^{K_A^{AGG}}}$$

$$PK_{A,t}^{AGG} \,.K_{A,t}^{AGG} = \sum_{capital} (rK_{capital\ ,A,t} \,.K_{capital\ ,A,t})$$

2.3 Commodity Production and Allocation

Production quantities, disaggregated by activity, are defined as yields times activity levels. On the left-hand side, these quantities are allocated to market sales and home consumption.

$$YAC_{A,C,t} + \sum_H CHA_{A,C,H,t} = \theta_{A,C} \,.\, YA_{A,t}$$

Output Aggregation Function: Aggregate marketed production of any commodity is defined as a CES aggregate of the marketed output levels of the different activities producing the commodity

$$YC_{C,t} = A_C^{ac} \,.\, \sum_A (b_{A,C}^{ac} \,.\, YAC_{A,C,t}^{-\sigma_C^{ac}})^{1-\sigma_C^{ac}}$$

First-Order Condition for Output Aggregation Function The optimal quantity of the commodity from each activity source is inversely related to the activity-specific price.

$$PYAC_{A,C,t} = PYC_{C,t} \,.\, YC_{C,t} \,.\, b_{A,C}^{ac} . YAC_{A,C,t}^{\;\;-\sigma_C^{ac}-1} . \sum_{AP} (b_{AP,C}^{ac} \,.\, YAC_{AP,C,t}^{-\sigma_C^{ac}})^{-1}$$

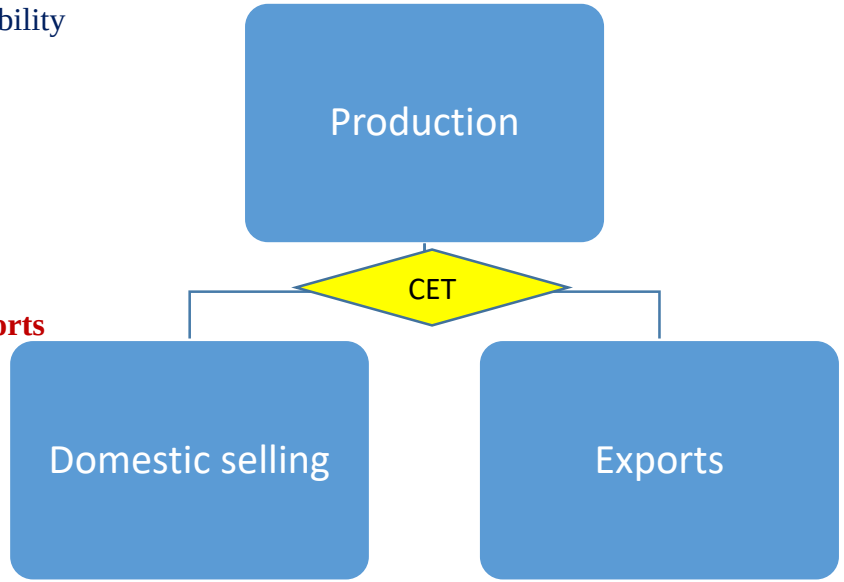
2.3. Exports vs Domestic supply

Output Transformation (CET) Function

The allocation of marketed domestic output to two alternative destinations: domestic sales and exports. With imperfect transformability between these two destinations.

$$YC_{C,t} = A_C^{ac} \cdot \sum_A (b_{A,C}^{ac} \cdot YAC_{A,C,t}^{-\sigma_C^{ac}})^{1-\sigma_C^{ac}}$$

$$YC_{C,t} = A_C^t \cdot (\sum_R (b_{C,R}^t \cdot EXP_{C,R,t}^{\sigma_C^t}) + \left(1 - \sum_R (b_{C,R}^t \cdot D_{C,t}^{\sigma_C^t}\right))^{\frac{1}{\sigma_C^t}}$$



Output Transformation for Domestically Sold Outputs and Exports

$$YC_{C,t} = D_{C,t} + \sum_R EXP_{C,R,t}$$

Export-Domestic Supply Ratio

Defines the optimal mix between exports and domestic sales

$$EXP_{C,R,t} = D_{C,t} \cdot (\frac{PEXP_{C,R,t}}{PDS_{C,t}} \cdot \frac{1-\sum_{RP} b_{C,R}^t}{b_{C,R}^t})^{1/(\sigma_c^t - 1)}$$

3. Demand

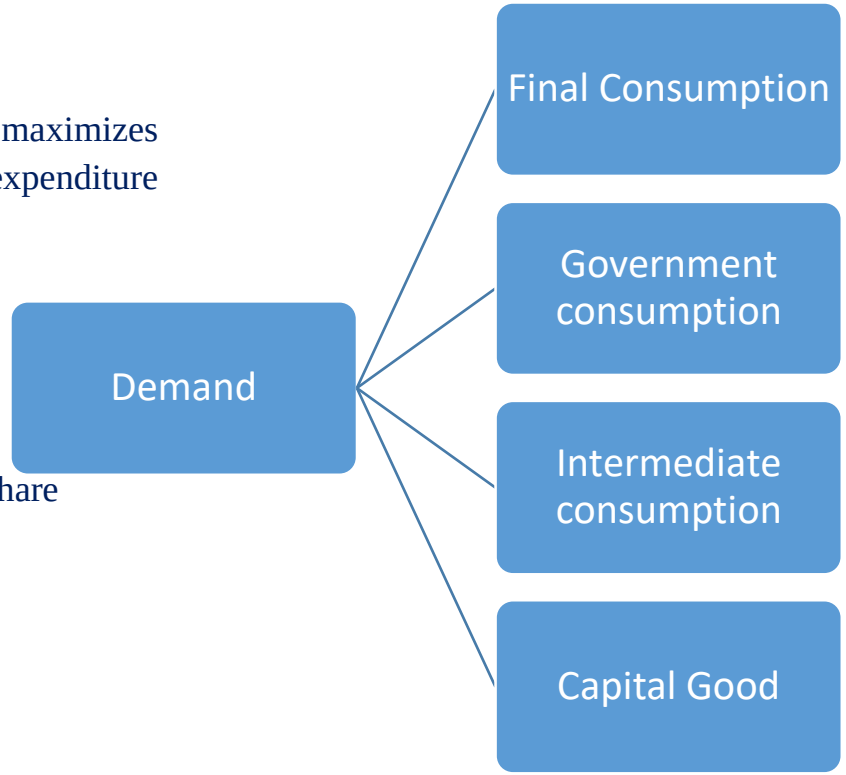
Inermediate consumption For each activity:

$$IC_{C,A,t} = ica_{C,A} \,.\, INT_{A,t}$$

Final consumption It is assumed that each household maximizes its utility function subject to a consumption expenditure constraint.

$$PDEM_{C,t}\, CH_{C,H,t} = PDEM_{C,t}\, \gamma^{m}_{C,H} + \beta^{m}_{C,H}(EXPH_{H,t} - \sum_{CP} PDEM_{CP,t}\, \gamma^{m}_{CP,H} - \sum_{A,CP} PYAC_{A,CP,t}.\gamma^{h}_{A,CP,H}$$

$$PYAC_{A,C,t}\,.CHA_{A,C,H,t} = PYAC_{A,C,t}.\gamma^{h}_{A,C,H} + \beta^{h}_{A,C,H}(EXPH_{H,t} - \sum_{CP} PDEM_{CP,t}.\gamma^{m}_{CP,H} - \sum_{AP,CP} PYAC_{AP,CP,t}.\gamma^{h}_{AP,CP,H}$$



Government Consumption is supposed to be a fixed share

$$PDEM_{C,t}\, \frac{G_{C,GOVF,t}}{EXPG_t - \sum_{INSDNG} trnsfr_{INSDNG,GOV,t}.CPI_t)} = \frac{PDEM0_C.G0_{C,GOVF}}{(EXPG0 - \sum_{INSDNG} trnsfr\,0_{INSDNG,GOV}.)}$$

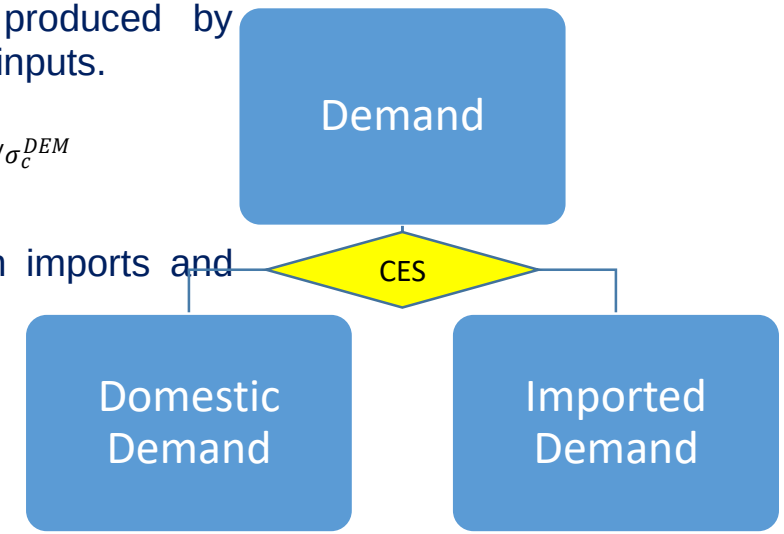
Capital Good Demand

$$\mathrm{KG(C,t)} = \mathrm{INVTOT(t)a_{INV\,(C)}}\left(\frac{\mathrm{PDEM(C,t)}}{\mathrm{PINVTOT(t)}}\right)^{\sigma_c^{INV\,(c)}}$$

3. Local VS Imported Demand

Imperfect substitutability between imports and domestic output sold domestically is captured by a CES aggregation function in which the composite commodity that is supplied domestically is produced by domestic and imported commodities entering this function as inputs.

$$DEM_{C,t}=A_c^{DEM}\left(\sum_R b_{C,R}^{DEM}\ IMP_{C,R,t}^{-\sigma_c^{DEM}}+\left(1-\sum_R b_{C,R}^{DEM}\right)D_{C,t}^{-\sigma_c^{DEM}}\right)^{-1/\sigma_c^{DEM}}$$



Import-Domestic Demand Ratio the optimal mix between imports and domestic output

$$\frac{IMP_{C,R,t}}{D_{C,t}}=\left(\frac{PD_{c,t}}{PIMP_{C,R,t}}.\frac{b_{C,R}^{DEM}}{1-\sum_{RP} b_{C,R}^{DEM}}\right)^{\frac{1}{1+\sigma_c^{DEM}}}$$

Demestic demad.

$$DEM_{C,t}=D_{C,t}+\sum_R IMP_{C,R,t}$$

Demand For Transactions Services Total demand for trade inputs is the sum of the demands for these inputs that are generated by imports, exports, and domestic market sales

$$TR_{C,t}=\sum_{CP} icd_{C,CP}.D_{CP,t}+\sum_{CP,R} icm_{C,CP}.IMP_{CP,R,t}+\sum_{CP,R} ice_{C,CP}.EXP_{CP,R,t}$$

4. Institution Block

Factor Income

$$YF(F,t) = \sum_{A} \mathrm{W(F,A,t)Q(F,A,t)}$$

Factor incomes to domestic institutions The income of each factor is split among domestic institutions in fixed shares after payment of direct factor taxes and transfers to the rest of the world.

$$YIF_{INSD,F,t} = shif_{INSD,F}.\left[\left(1 - tf_{f,t}\right).YF_{F,t} - trnsfr_{row,F,t}.EXR_t\right]$$

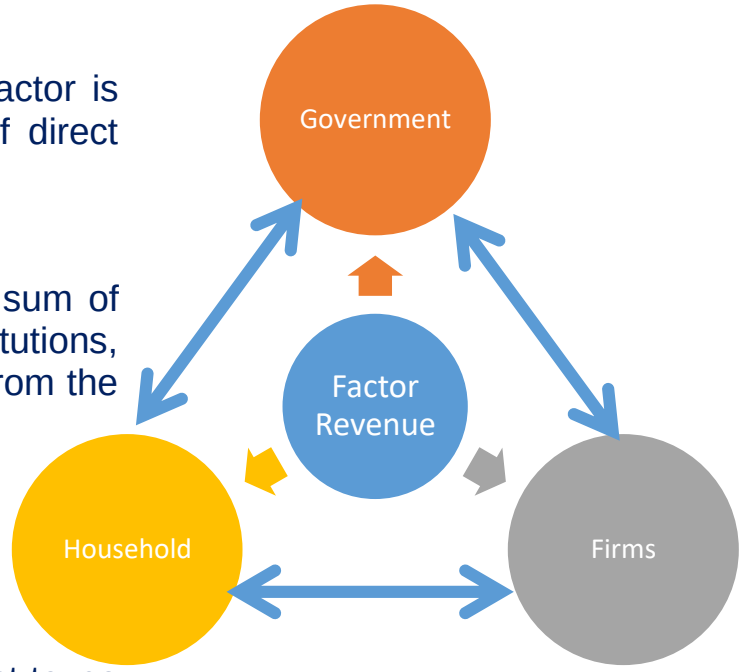
Total incomes of domestic nongovernment institutions is the sum of factor incomes, transfers from other domestic nongovernment institutions, transfers from the government (indexed to the CPI), and transfers from the rest of the world.

$$YI_{INSDNG,t} = \sum_f YIF_{INSDNG,F,t} + \sum_{INSDNGP} TRII_{INSDNG,INSDNGP,t} + trnsfr_{INSDNG,GOV,t}.CPI_t + trnsfr_{insdng,ROW,t}.EXR_t$$

Transfers to Institutions from Institutions

Transfers between domestic nongovernment institutions are paid as fixed shares of the total institutional incomes net of direct taxes and savings.

$$TRII_{INSDNG,INSDNGP,t} = shii_{INSDNG,INSDNGP}.\left(1 - to_{sav\ INSDNGP,t}\right).\left(1 - Tax_{Dir\ INSDNGP,t}\right).YI_{INSDNGP,t}$$



Government Account

Total Government Income Total government revenue is the sum of revenues from taxes, factors, and transfers from the rest of the world.

$$YG_t = \sum_{INSDNG} Tax\_Dir_{INSDNG,t}.YI_{INSDNG,t} + \sum_f tf_{F,t}.YF_{F,t} + \sum_A tva_{A,t}.PVA_{A,t}.VA_{A,t} + \sum_A ta_{A,t}.PYA_{A,t}.YA_{A,t} + \sum_{CM,R} tm_{CM,R,t}.pwm_{CM,R,t}.IMP_{CM,R,t}.EXR_t + \sum_{CE,R} te_{CE,R,t}.pwe_{CE,R,t}.EXP_{CE,R,t}.EXR_t + \sum_C TQ_{C,t}.PDEM_{C,t}.DEM_{C,t} + \sum_F YIF_{GOV,F,t} + trnsfr_{GOV,ROW,t}.EXR_t$$

Total Government current Expenditures: Current expenditure is supposed to be made of goods and services purchasing and transfers to non-governmental domestic institutions

$$EXPG_t = \sum_{C,GOVF} PDEM_{C,t}.G_{C,GOVF,t} + \sum_{INSDNG} trnsfr_{INSDNG,GOV,t}.CPI_t$$

Public investment: is supposed to be fixed (exogenous)

$$INVPUB(A,t) = \overline{INVPUB(A,t)}$$

Markets clearance 1

Composite Commodity Market Equilibrium: (Goods and Services market clearance) This equation imposes equality between quantities supplied and demanded of the composite commodity. The composite commodity supply, *DEM*, drives demands for domestic marketed output, *QD*, and imports, *QM*. The market-clearing variables are the quantities of import supply, for the import side, and the two interrelated domestic prices, *PDD* and *PDS*, for domestic market output.

$$DEM_{C,t} = \sum_A IC_{C,A,t} + \sum_H CH_{C,H,t} + \sum_{GOVF} G_{C,GOVF,t} + KG_{C,t} + qdst_{C,t} + TR_{C,t}$$

Current Account Balance for the Rest of the World: The current-account balance imposes equality between the country's spending and its earning of foreign exchange. For the basic model version, foreign savings is fixed; the (real) exchange rate (*EXR*) serves the role of equilibrating variable to the current-account balance. The fact that all items except imports and exports are fixed means that, in effect, the trade deficit also is fixed. Alternatively, the exchange rate may be fixed and foreign savings unfixed. In this case, the trade deficit is free to vary.

$$\sum_{CM,R} pwm_{CM,R,t}.IMP_{CM,R,t} + \sum_F trnsfr_{ROW,F,t} + \sum_{INSD} trnsfr_{ROW,INSD,t} +$$
$$INTF_t = \sum_{CE,R} pwe_{CE,R,t}.EXP_{CE,R,t} + \sum_{INSD} trnsfr_{INSD,ROW,t} + FSAV_t$$

Markets clearance 2

Government Balance The government balance imposes equality between current government revenue and the sum of current government expenditures (including interest payment) and savings

$$GSAV_t = YG_t - EXPG_t - INVPUB_{tot\ t} - INT_t$$

Savings-Investment Balance This equation states that total savings and total investment have to be equal. Total savings is the sum of savings from domestic nongovernment institutions, the government, and the rest of the world, with the last item converted into domestic currency. Total investment is the sum of the values of fixed investment (gross fixed capital formation) and stock changes. In the basic model version, the flexible variable, *to-sav*, performs the task of clearing this balance

$$FSAV_t.EXR_t = \sum_{FCAP,A} PINVTOT_t.INVP_{FCAP,A,t} + \sum_C PDEM_{C,t}.qdst_{C,t} - \sum_{INSDNG} to\_sav_{INSDNG,t}.\left(1 - Tax_{Dir\ INSDNG,t}\right).YI_{INSDNG,t} + GSAV_t + WALRAS_t$$

$$\sum_{INSDNG} to\_sav_{INSDNG,t}.\left(1 - Tax_{Dir\ INSDNG,t}\right).YI_{INSDNG,t} + GSAV_t + WALRAS_t + FSAV_t.EXR_t = \sum_{FCAP,A} PINVTOT_t.INVP_{FCAP,A,t} + \sum_C PDEM_{C,t}.qdst_{C,t}$$

Dynamics

Capital Accumulation

$$K(A,t) = (1-\delta)K(FCAP, A, t-1) + INV(A,t)$$

Labor supply

$$\mathrm{LS(SKL, t)} = \mathrm{LSl(SKL, t-1)}\ (1 + \mathrm{g_{L(t)}})$$