Life tables

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Outline

- The concept of a cohort life table
- Period life tables
- Issues in life table construction
- Model life tables
- Using MORTPAK

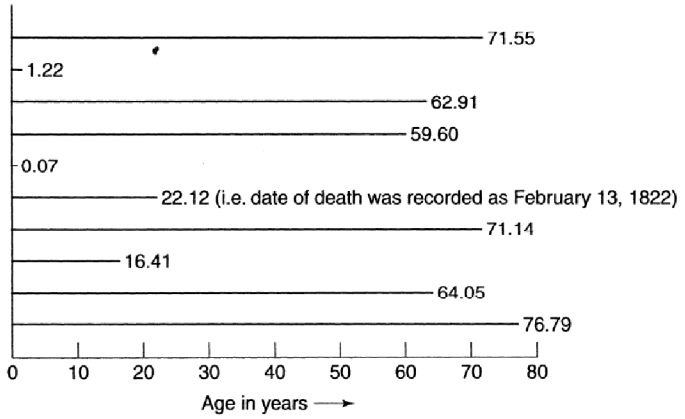


Figure 3.1 Age at death and life-lines of a hypothetical cohort of births (10 in all); date of birth: January 1, 1800

Life table columns

- x = exact age
- l_x = number left alive at age x
- ${}_nd_x$ = number dying between ages x and x+n
- ${}_nq_x$ = probability of dying between ages x and x+n
  - ${}_nq_x = {}_nd_x / l_x$

Life table columns

- ${}_nP_x$ = probability of surviving between ages x and x+n
  - ${}_nP_x = 1 - {}_nQ_x$
  - ${}_nP_x = l_{x+n} / l_x$
- ${}_nL_x$ = Person-years lived between ages *x* and *x+n*
- T_x = Person-years lived above age *x*

Life table columns

- e_x = Life expectancy at age *x*
  - $e_x = T_x / l_x$
- ${}_nm_x$ = death rate in the cohort between ages *x* and *x+n*
  - ${}_nm_x = {}_nd_x / {}_nL_x$
- ${}_na_x$ = average person-years lived in the interval by those dying in the interval

${}_nm_x \rightarrow {}_nq_x$ conversion formula

$$ {}_nq_x = \frac{n \cdot {}_nm_x}{1 + (n - {}_na_x)\ {}_nm_x} $$

Period life tables

- Concept of a synthetic cohort
- Based on observed the ${}_nM_x$ series for a specific period (like a calendar year) rather than on the ${}_nm_x$ series for a specific cohort
- Shows what *would* happen to a cohort *if* exposed to the age-specific mortality rates observed during a period
- Synthetic cohorts also called hypothetical or fictitious cohorts

Period life table construction

- Basic data
  - ${}_nD_x$ series for a calendar year
  - ${}_nN_x$ series for July 1 of that year
  - Traditional age breakdown for an *abridged* life table:
    - x = 0, 1, 5, 10, 15 …
    - Open-ended age interval starts at 85 or 100
  - Calculate ${}_nM_x = {}_nD_x / {}_nN_x$

Period life table construction

- Need to make assumption about ${}_na_x$ series
- Classic solution:
  - ${}_na_x$ = n/2 for x=5 and above
  - For open-ended age interval: $a_{85+} = 1 / M_{85+}$
  - For ${}_1a_0$ and ${}_4a_1$: use specific formulas

	Males	Females
	Value of ${}_1a_0$	
If ${}_1m_0 \geq .107$	.330	.350
If ${}_1m_0 < .107$	$.045 + 2.684 \cdot {}_1m_0$	$.053 + 2.800 \cdot {}_1m_0$
	Value of ${}_4a_1$	
If ${}_1m_0 \geq .107$	1.352	1.361
If ${}_1m_0 < .107$	$1.651 - 2.816 \cdot {}_1m_0$	$1.522 - 1.518 \cdot {}_1m_0$

Period life table construction

- Calculate ${}_nq_x$ column by applying ${}_nm_x \rightarrow {}_nq_x$ conversion formula, using
  - observed period ${}_nM_x$
  - assumed ${}_na_x$ series
- Last age group: $q_{85+} = 1$

Period life table construction

- Calculate ${}_np_x$ column
  - ${}_np_x = 1 - {}_nq_x$
- Calculate the l_x column
  - Start with $l_0 = 100{,}000$ (= *radix* of life table)
  - $l_{x+n} = l_x * {}_np_x$

Period life table construction

- Calculate ${}_nd_x$ column
  - ${}_nd_x = l_x * {}_nq_x$
  - Last age group: $d_{85+} = l_{85}$
- Calculate ${}_nL_x$ column:
  - ${}_nL_x = l_{x+n} * n + {}_na_x * {}_nd_x$
  - Last age group: $L_{85+} = l_{85} * a_{85+}$

Period life table construction

- Calculate T_x column starting with last age group
  - $T_{85} = L_{85+}$
  - $T_{80} = T_{85} + {}_5L_{80}$
  - $T_{75} = T_{80} + {}_5L_{75}$
  - …
- Calculate e_x
  - $e_x = T_x / l_x$

Example: Ireland 1986

TABLE 1
DATA

AGE AT LAST BIRTHDAY	ESTIMATED MID-YEAR POPULATION					BIRTHS BY AGE OF MOTHER AND SEX	DEATHS			AGE AT LAST BIRTHDAY
	BOTH SEXES	MALES		FEMALES			BOTH SEXES	MALES	FEMALES	
		Number	Percent	Number	Percent					
0	61172	31315	1.8	29857	1.7		534	295	239	0
1-4	262906	135103	7.8	127803	7.2		113	65	48	1-4
5-9	350650	179847	10.2	170803	9.6		84	56	28	5-9
10-14	349973	179381	10.1	170592	9.6	2	94	59	35	10-14
15-19	331100	169887	9.6	161213	9.1	2642	178	144	34	15-19
20-24	286424	144112	8.1	142312	8.0	11768	198	160	38	20-24
25-29	258439	129086	7.3	129353	7.3	19921	167	126	41	25-29
30-34	242689	122198	6.9	120491	6.8	16674	205	135	70	30-34
35-39	229740	116410	6.6	113330	6.4	8309	244	156	88	35-39
40-44	191751	97962	5.5	93789	5.3	1991	324	207	117	40-44
45-49	161740	82769	4.7	78971	4.5	114	551	321	230	45-49
50-54	147511	75156	4.2	72355	4.1	4	869	554	315	50-54
55-59	142215	70514	4.0	71701	4.0		1504	944	560	55-59
60-64	139978	67219	3.8	72759	4.1		2543	1603	940	60-64
65-69	129498	61080	3.5	68418	3.9		3661	2334	1327	65-69
70-74	110996	50881	2.9	60115	3.4		5279	3179	2100	70-74
75-79	75519	32635	1.8	42884	2.4		5788	3239	2549	75-79
80-84	42884	16126	0.9	26758	1.5	31756 M	5309	2507	2802	80-84
85+	25458	8009	0.5	17449	1.0	29669 F	5982	2187	3795	85+
TOTAL	3540643	1769690		1770953		61425	33627	18271	15356	TOTAL

Example: Ireland 1986

TABLE 2
MALE LIFE TABLE

x	$_nM_x$	$_nq_x$	l_x	$_nd_x$	$_nL_x$	$_nm_x$	$_na_x$	T_x	$\mathring{e}_x$	x
0	0.009420	0.009340	100000	934	99146	0.009420	0.086	7079886	70.799	0
1	0.000481	0.001922	99066	190	395788	0.000481	1.500	6980740	70.466	1
5	0.000311	0.001556	98876	154	493993	0.000311	2.500	6584952	66.598	5
10	0.000329	0.001643	98722	162	493258	0.000329	2.839	6090958	61.698	10
15	0.000848	0.004243	98560	418	491832	0.000850	2.690	5597700	56.795	15
20	0.001110	0.005541	98141	544	489359	0.001111	2.522	5105868	52.026	20
25	0.000976	0.004867	97597	475	486798	0.000976	2.496	4616510	47.302	25
30	0.001105	0.005513	97122	535	484309	0.001105	2.567	4129712	42.521	30
35	0.001340	0.006695	96587	647	481419	0.001343	2.655	3645402	37.742	35
40	0.002113	0.010592	95940	1016	477410	0.002128	2.744	3163984	32.979	40
45	0.003878	0.019372	94924	1839	470518	0.003908	2.769	2686574	28.302	45
50	0.007371	0.036396	93085	3388	457787	0.007401	2.745	2216056	23.807	50
55	0.013387	0.064958	89697	5827	435188	0.013389	2.718	1758269	19.602	55
60	0.023847	0.112931	83871	9472	397175	0.023847	2.658	1323081	15.775	60
65	0.038212	0.176100	74399	13097	340920	0.038212	2.615	926906	12.446	65
70	0.062479	0.271067	61372	16636	266264	0.062479	2.560	584986	9.532	70
75	0.099249	0.397860	44736	17799	178820	0.099534	2.480	318722	7.125	75
80	0.155463	0.552836	26937	14892	95791	0.155463	2.388	139902	5.194	80
85	0.273068	1.000000	12045	12045	44111	0.222183	3.662	44111	3.662	85

Illustration: Sweden, females

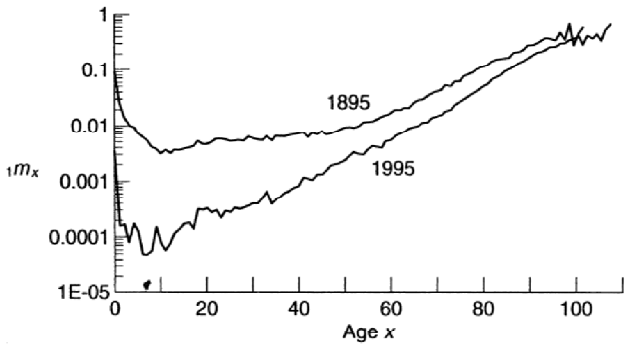


Illustration: Sweden, females

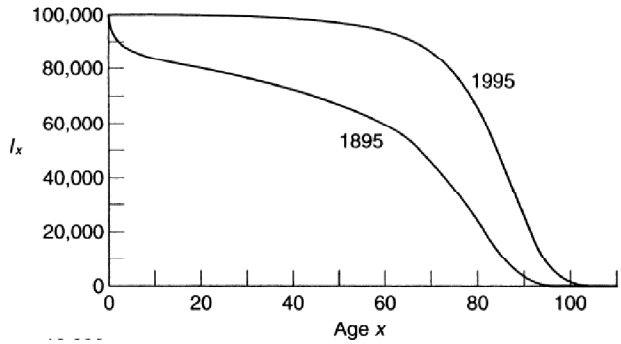


Illustration: Sweden, females

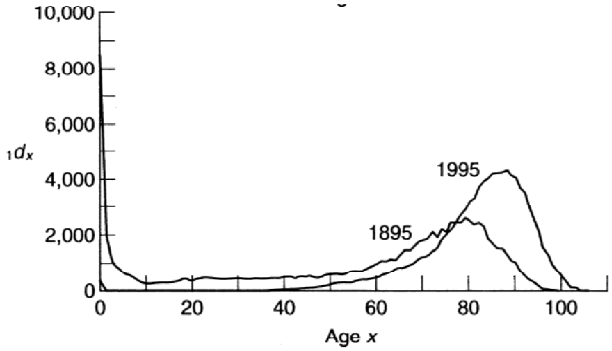
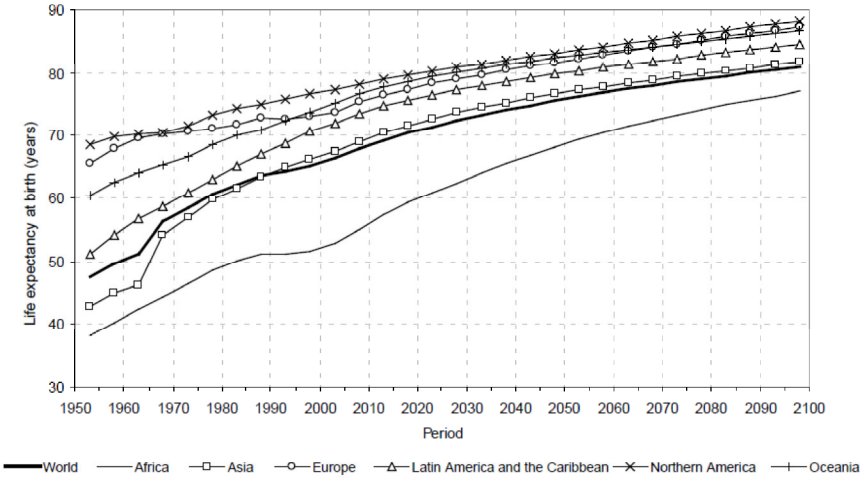


Figure 5. Life expectancy at birth for the world and the major areas, 1950-2100



Source: Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat (2011). World Population Prospects: *The 2010 Revision*. New York: United Nations.

Model life tables

- Model life tables useful for:
  - estimating mortality when incomplete/faulty data
  - modeling demographic processes
  - projections

Coale and Demeny (1966 and 1983)

- Based on 326 male and 326 female life tables
- Mostly 19th and first half of 20th century
- Data quality checks: only 192 life tables were retained
- Four "families" of life tables identified, based on geographical clustering

  * North * East * South * West

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Coale and Demeny (1966 and 1983)

- Life tables ranked by mortality "levels"
- To higher mortality levels (from 1 to 25) correspond higher levels of life expectancy (from 20 to 80 years)
- Two-entry system (1 family + 1 mortality index)
- Cautionary note
  - Not very adequate for low mortality
  - Low-income countries not represented in the database

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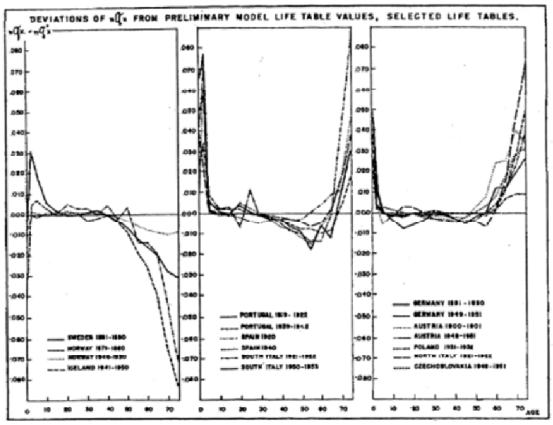
TABLE I. Distribution of 326 selected life tables according to year of origin and geographic location

Continent	Midpoint of reference period of life table				
	Before 1870	*1871–1918*	*1919–1945*	*After 1945*	*Total*
Africa			5	10	15
America, North		3	7	8	18
America, Latin		7	10	16	33
Asia		7	9	16	32
Europe	23	63	62	58	206
Oceania		10	6	6	22
Total	23	90	99	114	326

SOURCE: COALE & DEMENY, 1983.

Figure 9.1
Typical Deviations with Age in the North, South and East Regional Models



Source: Coale and Demeny (1983: 11)

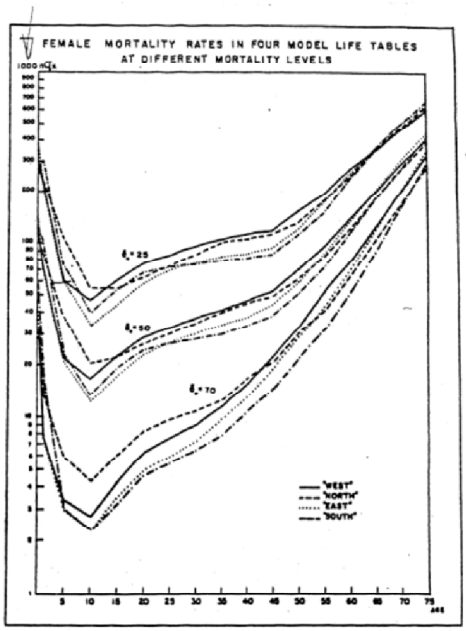


FIGURE 3. Values of $_nq_x$ in the four families of model life tables ("West," "North," "East," and "South") when e_0 = 25, 50, and 70 years, for females.

(39)

Infant and Child Mortality Rates for Females by Life Expectancy at Birth and Region (per thousand)

	e^0_0 30.00		40.00		50.00		60.00		70.00	
Model	$_1q_0$	$_4q_1$	$_1q_0$	$_4q_1$	$_1q_0$	$_4q_1$	$_1q_0$	$_4q_1$	$_1q_0$	$_4q_1$
West	256.1	177.9	178.2	118.3	118.8	72.3	71.2	33.5	31.1	7.8
North	224.3	203.8	156.9	137.6	106.0	85.8	66.3	43.2	32.6	13.8
East	306.5	169.2	216.8	113.2	147.4	66.2	89.7	31.6	40.9	7.6
South	228.8	250.0	172.5	166.5	131.0	100.0	94.9	50.0	59.1	17.2

Source: Coale and Demeny (1983).

U.N. life tables for developing countries (1982)

- Based on data from Africa, Asia, and Latin America
- Five "families"
  - Latin American
  - Far Eastern
  - South Asian
  - Chilean
  - General
- To select model life tables
  - Look at tables printed in books (indexed by e_0 and family)
  - Use software MORTPAK

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Using MORTPAK

1. Choose the “MATCH” procedure by selecting “Application” and then “MATCH” from the drop down menu
2. Select Sex (males or females)
3. Select Model life table pattern
4. Type in your input life table value in the cell “has value of”
5. Choose the column of the life table to which your input value pertains (${}_nm_x$, ${}_nq_x$, e_x or l_x)
6. Type in the age (beginning of interval) to which your input value pertains
7. Click on “Run”