اللجنة الاقتصادية والاجتماعية لغربي آسيا

Statistical tests (Redundancy &Robustness) - MPI

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Summary

If the aim of the poverty analysis is to have a robust and strong policy measure, then the initial assumptions (normative or else) that are taken in the AF method to produce those measures need to be tested:

❖Choice related to the framework, Indicators/Dimensions – Redundancy

❖Choice related to MPI calculation, Normative assumptions (Deprivation & poverty cut-offs, weights) – Robustness

❖ Measuring the degree of uncertainty (survey accuracy) in MPI – Confidence interval (Part of the Robustness)

Redundancy Test

- Analyzes the interaction and association between indicators
- Helps determining next action of combining or excluding indicators, or their categorization.
- Is only **informative**, and normative decisions are to be taken by policy makers.
- The R0 test results will help us to answer the following question: When analyzed independently, which of the indicators convey the same message as other indicator analysis, in that case, shall we remove/ retain one of them?

Example on Redundancy Test

$$R^0 = \frac{percentage\ of\ people\ deprived\ in\ Ind1\ and\ Ind2}{\min(H_{Ind1}, H_{Ind2})} \in [0,1]$$

Proportion of deprivation matches in the lowest level as a proportion of the minimum of the marginal deprivation rates.

		Ind 1 *(Access to Health Insurance)*		
		Non deprived	Deprived	Total
Ind 2 (Access to medicine)	Non deprived	*12.11%*	*11.96%*	*24.07%*
	Deprived	*28.13%*	***47.8%***	*75.93%*
	Total	*40.24%*	*59.76%*	*100%*

$$R^0 = \frac{47.8}{\min(59.76, 75.93)}$$

$$= 0.8^*$$

It seems that both indicators are somewhat highly associated, but what shall we do in that case

- Drop one of them?
- Combine them?
- Adjust their grouping into dimensions?

**0 means no association*
1 is high association

Example on redundancy test – What about other indicators

Redundancy matrix of indicators

	$Ind1$	$Ind2$	$Ind3$
$Ind1$	1		
$Ind2$	0.80	1	
$Ind3$	0.21	0.42	1

Ind 1 : Access to Health Insurance

Ind 2: Access to medicine

Ind 3: Water & Sanitation

- Arguments for keeping Vs. Arguments for eliminating

Low association (low redundancy) : keep indicators if each is highly informative independently - Retain low associated indicators (1&3) as they are important independently

High association (high redundancy): May keep indicators for normative purposes or because their correlation may change over time – Monitoring purposes.

- *Equally important is to think even more, and analyze if the same message is conveyed disaggregated by groups, although at the National level, it seems that a big percentage of the people who are deprived in one indicator are also deprived in another indicator, but is that true for all regions?*

- *Removing an indicator just because of its high association with another indicator is removing all other associations/relationships with the other indicators. Therefore, a global view is to be considered for each indicator instead of pairwise comparison to prevent any possible loss of information.*

B- Robustness

❖As designing MPI involves variation in relative indicator weights, and/or poverty/ deprivation cut-offs, it is important to create a robust MPI measure.

❖Since Headcount (H) and Intensity (I), and thus MPI, are sensitive to these changes, it is important to consider how these changes alter the ranking of states/ Governates/ Provinces (or any population subgroups) within a country and the composition of poverty (i.e. demographic sub-group).

❖Why demographic sub-group is preferred? Normally, budget allocation to the poor in a country is distributed according to MPI rank. Changing normative assumptions, will induce a change in MPI, and this can change the ranking of groups.

B- Robustness

❖Question that need to be addressed by Robustness: Will the order of the regions by the scale of how poor they are stays the same when different assumptions are given?

❖How many flips and how sensitive are they? If no changes are incurred, this simply means that our results are robust.

❖*Recap*

“H” is the percentage of the people that are multidimensionally poor

“I” reflects the intensity of the poor (from those poor people, on average, in how many indicator are they deprived). The higher the intensity the more deprived these poor people are.

“MPI” is the percentage of deprivations poor people experience, as a share of the possible deprivations that would be experienced if all people were deprived in all indicators at the same time.

B- Assumption 1 : Deprivation Cut-Off

Deprivation cut-off is the minimum level required to consider a household/individual deprived in this indicator/dimension

Deprivation cut-offs are normative decisions subject to international standards, national targets…

Example:

In the dimension “Education”, the indicator “Years of Schooling” has a cut-off of 6. A household is deprived if s/he has not completed six years of schooling. Completing only three years of schooling will consider the household as deprived in this dimension.

	$Ind1$	$Ind2$	$Years\ of\ schooling$
A	19	1	7
B	12	1	**3**
C	15	**0**	**1**
D	**12**	**0**	**0**
$cut\ off$	13	1	6

1 if deprived
0 if not

	$Ind1$	$Ind2$	$Years\ of\ schooling$
A	0	0	0
B	0	0	1
C	0	1	1
D	1	1	1

B- Assumption 2 : Indicators Weights

Defining a weight for each indicator/dimension illustrates the relative importance of each indicator/dimension in the final MPI measure.

Weights are fixed over time, and they are usually set by policy makers according to their target.

Intuitively, different weights will lead to different MPI results.

Non-Weighted censored matrix

Weighted censored matrix

Scenario 1

	$Ind1$	$Ind2$	$Ind3$
A	0	0	0
B	0	0	1
C	0	1	1
D	1	1	1
$weights$	1/2	1/3	1/6

	$Ind1$	$Ind2$	$Ind3$	$score$
A	0	0	0	0
B	0	0	1/6	1/6
C	0	1/3	1/6	1/2
D	1/2	0	1/6	2/3

MPI = 0.44

Scenario 2

	$Ind1$	$Ind2$	$Ind3$
A	0	0	0
B	0	0	1
C	0	1	1
D	1	1	1
$weights$	1/3	1/3	1/3

	$Ind1$	$Ind2$	$Ind3$	$score$
A	0	0	0	0
B	0	0	1/3	1/3
C	0	1/3	1/3	2/3
D	1/3	0	1/3	2/3

MPI = 0.55

B- Assumption 3 : Poverty Cut-Off

Poverty cut-off (*k*) identifies individuals/households who are multi-dimensionally poor in at least k weighted indicators.

An individual/household is considered poor if he/she scores more than *k**

Level of poverty cut-off are also subject to normative decisions and change according to the country's objectives

Different cut-offs <u>may</u> lead to different MPI measures, but it is not always the case.

* *A household is deprived in a dimension 1 if s/he scores less than this dimension's deprivation cut-off.*
A household is poor if s/he scores more than the poverty cut-off

Non-Weighted censored matrix

Weighted censored matrix

Scenario 1
K=1/3

	$Ind1$	$Ind2$	$Ind3$	$score$
A	0	1/3	0	1/3
B	0	0	1/6	1/6
C	0	1/3	1/6	1/2
D	1/2	0	1/6	2/3

→

	$Ind1$	$Ind2$	$Ind3$	$score$
A	0	1/3	0	1/3
B	0	0	0	0
C	0	1/3	1/6	1/2
D	1/2	0	1/6	2/3

MPI = 0.75

Scenario 2
K=1/2

	$Ind1$	$Ind2$	$Ind3$	$score$
A	0	1/3	0	1/3
B	0	0	1/6	1/6
C	0	1/3	1/6	1/2
D	1/2	0	1/6	2/3

→

	$Ind1$	$Ind2$	$Ind3$	$score$
A	0	0	0	0
B	0	0	0	0
C	0	1/3	1/6	1/2
D	1/2	0	1/6	2/3

MPI = 0.583

When the poverty cut-off increases (decreases), less (more) people will be identified as MPI poor.

B- Assumption 3 : Poverty Cut-Off

We can analyze robustness using

Robustness on a restricted set (Rank correlation pairwise comparison)

B.1

Kendall's Coefficient

Spearman Coefficient

Robustness on Continuous case

B.2

First-Order Stochastic Model

B.1 – Pairwise comparison: Rank Correlations Test - *Kendall's τ rank coefficient*

The aim is to assess how stable is the ranking of different regions or groups according to changes in *k (or any of the assumptions).*

Kendall's τ rank coefficient

$$R^{\tau} = \frac{(\# of\ Concordant^{*} - \# \ of\ discordant\ pairs)}{nC2} \in [-1,1]$$

The coefficient measures the number of concordance pairwise comparisons against discordance among all possible pairwise comparison available.

- R^{τ}= 1 means that ranks are perfectly positively associated with each other (*all concordant)*
- R^{τ}= -1 means ranks are perfectly negatively associated with each other (*all discordant)*

The closer to unity, the better, since it means that the variation did not have a large impact on MPI ranking.

* *The comparison between a pair of regions/ sub-regions is concordant if the poverty ordering is preserved, discordant if not.*

B.1 – Pairwise comparison: Rank Correlations Test - *Kendall's τ rank coefficient*

The analysis can be made on the difference assumptions and different poverty measures as well. The below example is computed on different poverty cut-off assumptions and on MPI. An alternative ranking could have been done on intensity for instance.

Region	MPI *(K=1/3)*	Rank	MPI *(K=2/3)*	Rank
A	0.420	2	0.379	3
B	0.645	1	0.54	1
C	0.320	3	0.40	2
D	0.289	4	0.356	4

Is A(or B) poorer than B (or A) in both scenarios?

	A	B	C	D
A	–	–	–	–
B	1	–	–	–
C	0	1	–	–
D	1	1	1	–

Concordant = 5
Discordant = 1
4C2=6
T = (5-1)/6 = 66.66%

τ
66.66%

** Concordant: rank does not change*

B.1 – Pairwise comparison: Rank Correlations Test - *Spearman's ρ rank coefficient*

$$\rho = 1 - \frac{6 * \sum_{i=1}^{n} r_i}{n(n^2 - 1)} \in [-1,1]$$

r_i is the difference in the ranking of Region I that is exhibited in both scenarios, n is the number of regions. The closest to 1 the better, since it means that ranking is more stable. The coefficient assesses monotonic pairwise

Region	MPI (K=1/3)	Rank	MPI (K=2/3)	Rank
A	0.420	2	0.379	3
B	0.645	1	0.54	1
C	0.320	3	0.40	2
D	0.289	4	0.356	4

$$\rho = 1 - \frac{6 * [(3-2)^2 + (1-1)^2 + (2-3)^2 + (4-4)^2]}{4(4^2 - 1)}$$

$$= 0.8$$

ρ
80%

Summary of both robustness methods mentioned above B.1

- Kendall's τ rank coefficient establishes stability of **relative** poverty orderings between pairs
- Spearman ρ rank coefficient establishes stability of **absolute** position of each region
- It is important to note that point estimate comparison is not enough to rank regions (look for Regions A&D with small MPI difference). We need to consider confidence intervals as mentioned before.

Confidence Intervals

<u>How conclusive these comparisons are?</u>

Instead of using a point estimate MPI for these comparisons and rankings, we must consider MPI with its upper and lower bounds. Thus, it is crucial to compute a measure of confidence or reliability for each estimate from a sample survey, is this survey representative? Is the sample reliable? The lower the magnitude of a standard error, the larger the reliability of the corresponding estimate.

<u>When to apply it and how?</u>

Standard errors are key for hypothesis testing and for the construction of confidence intervals, both of which are very helpful for robustness analysis and more generally for **drawing policy conclusions.**

MPI CI values can be estimated from household surveys that are subject to uncertainty and errors.

Confidence Intervals

We consider and MPI estimation using w =95% confidence level for a fixed poverty cut-off *k*

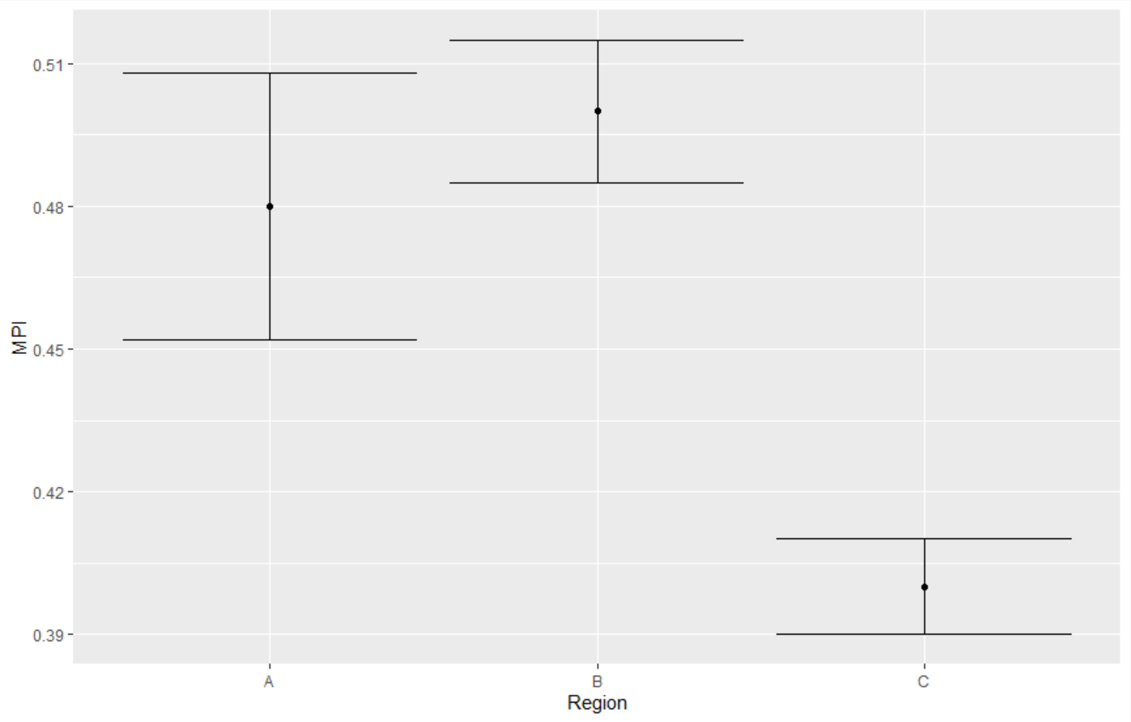
Region	MPI (*k*=28%)	MPI Upper and lower bound
A	0.48	0.452 0.508
B	0.5	0.485 0.515
C	0.4	0.39 0.41

$$\textit{Confidence Interval} = MPI\ estimate\ \pm t_{(1-\frac{w}{2},df)} * se_{MPI}$$

$$Where\ se_{MPI}\ is\ the\ sampling\ standard\ error$$

Confidence Intervals

MPI Confidence interval at 95% for three regions



❖ Regions A&B:

Confidence interval overlap so we cannot conclude that B is poorer than A.

$\Rightarrow$ Conduct Hypothesis testing

$$H_0: MPI_A = MPI_B$$

$$H_1: MPI_A < MPI_B$$

Rejecting H0 allows us to conclude that B is poorer than A

❖ Regions A&C:

Confidence interval do not overlap

$\Rightarrow$ A is poorer than C

❖ Regions B&C:

Confidence interval do not overlap

$\Rightarrow$ B is poorer than C

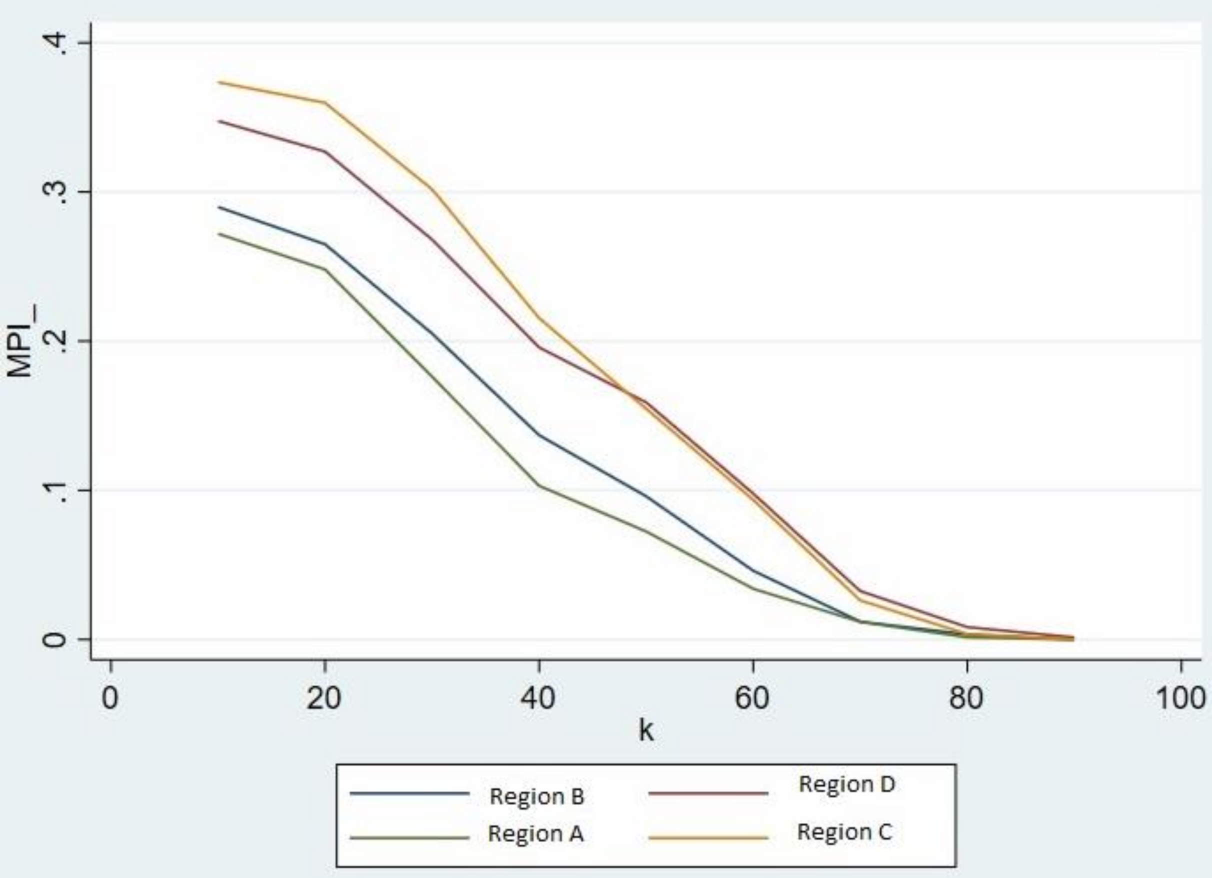
B.2 – Continuous set - Robustness Analysis

- First-order stochastic dominance (FSD) can be used to evaluate the sensibility of any pairwise combination (i.e. any two regions, or age groups) with respect to the poverty cut-off.

- Instead of the discrete points analysis, this will be a **continuous set** analysis.

- A possible result of such an analysis is to examine if multidimensional poverty in one region "dominates" the level of poverty of another region, regardless of the poverty cut-off used to compute the national MPI, otherwise know as: First order stochastic dominance.

B.2 – First Order Stochastic Dominance

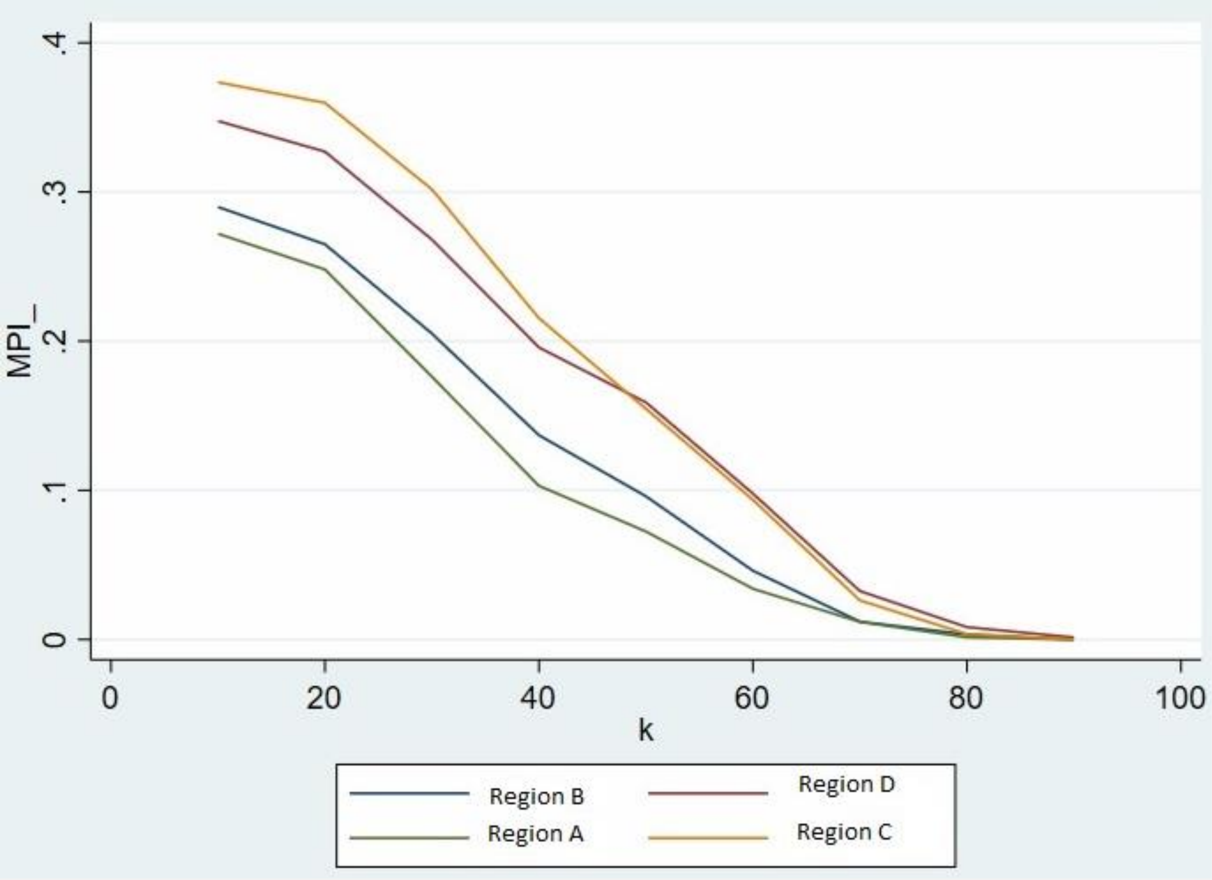


Region A's curve is permanently lower than other curves →
No matter the poverty line chosen, Region A ***is less poor*** than all other regions.

Similarly, Region B is ***less poor*** than Regions C&D $\forall k$

Region D's curve is lower than Region C for k$< 40\%$ →
Region D ***is less poor*** than Region C only in this interval

B.2 – First Order Stochastic Dominance



The dominance requirement for all possible poverty cutoffs may be an excessively stringent requirement. Practically, one may seek to verify the unambiguity of comparison with respect to a limited variation in the poverty cutoff, which can be referred to as restricted dominance analysis.

The robustness of pairwise comparisons for all poverty cut-offs that are found in the following interval $k \in [0, 0.4]$, for instance.